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# THE SELF-SIMILAR PROBLEM OF THE TRANSFER OF SAND FROM A BED INTO A WELL†

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The solution of the problem of the start-up of a well in a saturated stratum under conditions when sand production can occur is constructed. The matrix of the stratum breaks down and is transported in a granulated state when its yield point is reached. The flow of sand as a continuous medium occurs under the action of seepage forces and is described by the non-associated law of plasticity when the Mohr – Coulomb condition is satisfied. If the inflow of fluid to the well is constant and the initial state corresponds to the state of rest, the problem is found to be self-similar. Cases of an incompressible fluid (water or petroleum) and a gas are considered. Characteristic relations between the radii of the caverns which are formed and regions of plasticity and drainage are obtained. It is shown that the cohesion and dilatancy of the moving mass of sand are the key parameters determining the ratio of the flows of sand and fluid. 

Interest in the problem of the failure of saturated porous media under the action of seepage forces is primarily associated with the development of deep, rich hydrocarbon strata. The fact is that the high initial pore pressure is caused by the formation of a deposit without any drainage and, as a consequence, with a low strength of its matrix. As a result, even relatively small flows lead to the transfer of the fragmented rock (sand) into the well. When the strata occur at a shallow depth, the transfer of sand is typical of petroleum deposits of high viscosity. Quicksand phenomena are also known when the sand fills the well under the action of the initial stresses and impedes the boring of wells.

As is customary, the mathematical modelling of the failure of geomaterials and their conversion into a mobile state has to be carried out within the framework of the theory of elastoplasticity. It is well known that the specific details of the plastic behaviour of geomaterials results from their primordially polycrystalline structure with relatively weak bonds between the mineral grains. As a result, failure occurs under conditions when the forces of dry friction and adhesion are overcome and is accompanied by the growth of voids and microfissures (dilatancy) and their decrease or closure (which is taken into account by a change in sign of the dilatancy) An adequate mathematical model was first formulated within the framework of non-associated plastic flow [1] and, subsequently, in a simpler deformation form in [2].

In the case of the geomaterial which is saturated with fluid, it is necessary to use Biot's "poroelasticity" model [3]. However, in the case of flow processes, it is much more convenient to use a system of equations in the velocities [4, 5] relating, in particular, the rates of deformation with the displacements using Oldroyd derivatives.

The axisymmetric problem of the steady-state inflow of a fluid with sand into wells [6, 7] and spherically symmetric flow to an individual perforated hole in the column wall [8] have been treated in the above-mentioned formulation. It was found that the appearance of sand in subterranean flow is determined by the initial cohesion, but the quantitative relations are determined by the cohesion in the moving mass of sand and the coefficient of dilatancy. It was assumed in another investigation [9] that the moving fractures matrix is incompressible and porosity changes only occur in a stepwise manner in the fracture front.

Steady-state solutions in seepage theory diverge at infinity and are therefore only applicable in the case of a bounded flow domain. Correspondingly, the concept of a feeding contour (the radius of influence) of a well is introduced in the case of an infinite stratum. Under conditions where sand is transferred, it is necessary to invoke the hypothesis that the feeding contour and the external boundary of the plastic state are identical, in order to preserve the steady-state character of the flow. However, under unsteady-state conditions, these boundaries do not coincide and the solutions at infinity turn out to be bounded.

Self-similar solutions of the problem of the start-up of wells in infinite water (petroleum)-bearing and gas beds are proposed in this paper. All the boundaries, which are identical at the initial instant

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of time, subsequently move at different velocities, although proportionally to the square root of the time. The relations between the boundaries thereby turn out to be fixed. In all the solutions, the margin of the initial elastic energy of compression of the infinite bed serves as the source of the excess mass of the solid material (sand).

## 1. THE INITIAL SYSTEM OF EQUATIONS

The system of equations of motion includes the mass balances for the solid and fluid phases which, in the axisymmetric case, have the form

$$\frac{\partial}{\partial t}(m\rho_f)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho_f mw)=0,\quad \frac{\partial}{\partial t}(1-m)\rho_s+\frac{1}{r}\frac{\partial}{\partial r}(r\rho_s(1-m)v)=0 \tag{1.1}$$

where $m$ is the porosity and $w$, $v$, $\rho_f$, $\rho_s$ are the actual velocities and densities of the phases respectively.

When inertial forces are neglected, the law of conservation of momentum is identical to Darcy's law but is written for the relative velocity of the fluid flow

$$m(w-v)=-\frac{k}{\mu}\frac{\partial p}{\partial r} \tag{1.2}$$

Here $k$ is the permeability, $u$ is the viscosity of the fluid and $p$ is the pore pressure. The effective radial and tangential stresses $\sigma_r$, $\sigma_\theta$ (according to Terzaghi [11]) are found from the equation

$$\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}-\frac{\partial p}{\partial r}=0 \tag{1.3}$$

In the main part of it, the bed is in an elastic state. Here, the equations for the equilibrium of the medium become

$$\frac{\partial\sigma_r}{\partial r}+\frac{2G}{r}\left(\frac{\partial u}{\partial r}-\frac{u}{r}\right)-\frac{\partial p}{\partial r}=0 \tag{1.4}$$

where the well-known laws of poroelasticity [3–5]

$$\sigma_r=(K+G)\frac{\partial u}{\partial r}+(K-G)\frac{u}{r}+\varepsilon p$$
$$\sigma_\theta=(K-G)\frac{\partial u}{\partial r}+(K+G)\frac{u}{r}+\varepsilon p \tag{1.5}$$

have already been used.

Here, $u$ is the displacement, $\varepsilon=\beta_s K$ is the ratio of the compressibility $\beta_s$ of the grains of the solid material to the compressibility of the matrix $1/K$, and $G$ is the rigidity of the matrix. The displacements are related to the velocities in the usual way

$$\frac{\partial u}{\partial t}=v \tag{1.6}$$

Since, in the plane formulation of the problem (as in the spherical case) the unloading of the primordially elastically compressed bed serves as the source of the excess mass of sand, we will consider the laws of pressure redistribution in the external zone of the bed.

The equations of state of the phases are defined [4, 5] as follows:

$$\frac{\rho_f}{\rho_f^0}=1+\beta_f(p-p_0),\quad \frac{\rho_s}{\rho_s^0}=1+\beta_s(p-p_0)-\beta_s(\sigma-\sigma_0)/(1-m_0) \tag{1.7}$$

where $\beta_f$ is the compressibility of the fluid and $\sigma$ is the mean stress. We specify the equation of state of the gas as $\rho/\rho_0=p/p_0$.

If the effective elastic stresses acting in the matrix of the bed reach the critical level

$$(\sigma_r - \sigma_0)\theta_\sigma = -\alpha(\sigma_r + \sigma_\theta) + Y, \quad \theta_\sigma = \operatorname{sgn}(\sigma_r - \sigma_\theta) \tag{1.8}$$

failure of the medium occurs and plastic flow arises. Here $Y$ is the cohesion and $\alpha$ is the coefficient of internal friction. Plastic flow, which occurs beyond the disintegration front and which is localized in the neighbourhood of the well, takes place when a flow condition of the same form as in (1.8) is satisfied but with values of $Y_r$, $\alpha_r$ corresponding to the granulated state of the matrix.

The volume changes during the course of the plastic flow are proportional to the increments in the shear (modulus), which can be formulated in the form of a kinematic relation between the invariants of the displacement velocity field [4, 5]. In the axisymmetric version, the dilatancy relation is

$$\frac{\partial v}{\partial r} + \frac{v}{r} = \Lambda\theta\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right), \quad \theta = \operatorname{sgn}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) = \theta_\sigma \tag{1.9}$$

Here $\Lambda$ is the rate (coefficient) of dilatancy and the sign of the direction of the shear $\theta$ is taken into account. As a consequence of the coaxial form of the stress and rate of deformation tensors (of the positive dissipation of mechanical energy), this sign is the same as the sign in (1.8).

## 2. THE ELASTIC OUTER ZONE

The changes in the porosity $m$ can be eliminated from mass balance equations (1.1). In linearized form, we then obtain

$$\beta\frac{\partial p}{\partial t} = \varepsilon\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\{r[m_0 w + (1 - m_0)v\ ] \tag{2.1}$$

$$\beta = m_0\beta_f + (1 - m_0)\beta_s - \varepsilon\beta_s, \quad \varepsilon = K\beta_s$$

if the relation

$$\frac{\partial \sigma}{\partial t} = K\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \varepsilon\frac{\partial p}{\partial t}$$

which follows from relations (1.5) and (1.6) and the equations for the compressibilities of phases (1.7) are used. Here $\beta$ is the overall compressibility of the geomaterial of the bed. We rewrite this equation as follows

$$\beta\frac{\partial^2 p}{\partial r \partial t} - \frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}[r(m_0 w + (1 - m_0)v\ )]\right\} = \varepsilon\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv\ )\right] \tag{2.2}$$

On the other hand, since

$$e = \frac{\partial u}{\partial r} + \frac{u}{r} = \frac{1}{r}\frac{\partial}{\partial r}(ru), \quad e_r = \frac{\partial u}{\partial r}, \quad e_\theta = \frac{u}{r}$$

elastic equilibrium equation (1.4) is transformed, by virtue of laws of poroelasticity (1.5), to the form

$$\frac{\partial p}{\partial r} = \frac{K + G}{1 - \varepsilon}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right]$$

When $p_0 = p_\infty = \text{const}$, integration of this equation with respect to the radius leads to the relation

$$p - p_0 = \frac{K + G}{1 - \varepsilon}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv\ )\right] \tag{2.3}$$

and subsequent differentiation with respect to time gives

$$\frac{1 - \varepsilon}{K + G}\frac{\partial p}{\partial t} = \frac{\partial v}{\partial r} + \frac{v}{r} \tag{2.4}$$

Substituting relation (2.4) into Eq. (2.1) and using equality (1.2), we obtain the classical piezoconduction equation

$$\frac{\partial p}{\partial t} = \varkappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right], \quad \varkappa = \frac{\varkappa_1\varkappa_2}{\varkappa_1(1-\varepsilon)+\varkappa_2} \tag{2.5}$$

which must be solved for the outer zone of the bed $(r, \infty)$.

Note that Eq. (2.4) can be written in the like form

$$\frac{k}{\mu}\frac{\partial p}{\partial t} = \varkappa_1 \frac{1}{r}\frac{\partial}{\partial r}(rv), \quad \varkappa_1 = \frac{k}{\beta\mu}$$

If $\varepsilon \ll 1$, the estimate of the piezoconduction coefficients $\varkappa_1 \ll \varkappa_2$ holds and the outer zone will be characterized by a smaller piezoconductance.

$$\varkappa_2 = \frac{k}{\mu}\frac{K+G}{1-\varepsilon}$$

We emphasize that the piezoconduction coefficient is simple to determine by tests of the wells which also enables one to connect the analysis to the conditions found in practice.

The pressure field which arises during the start-up of a well with a constant output in an infinite bed corresponds, as is well known, to the classical solution of Eq. (2.5)

$$p(\xi) = p_\infty - \frac{Q_0\mu}{4\pi kh}\int_{\xi^2/4}^{\infty}\frac{\exp(-\eta)}{\eta}d\eta, \quad \xi = \frac{r}{\sqrt{\varkappa t}} \tag{2.6}$$

However, $Q_0$ now has the meaning of the output of only the fluid from a certain "effective" well which replaces the real well into which a mixture of sand and fluid enters. Moreover, here $h$ is the bed thickness and the self-similar variable $\xi$ has appeared, which we shall subsequently use.

The rate of displacement field, which vanishes at infinity and corresponds to (2.6), is obtained by the integration of Eq. (2.4) with respect to the radius

$$v = \frac{Q_0}{4\pi h\varkappa_2 r}\int_r^{\infty} r\exp\left(-\frac{r^2}{4\varkappa t}\right)dr + \frac{A}{r} \tag{2.7}$$

The linearization which has been used enables us to find the porosity distribution from the mass balance of the solid phase

$$\frac{\partial m}{\partial t} = (1-m_0)\frac{\partial}{\partial t}\frac{\rho_s}{\rho_s^0} + (1-m_0)\frac{k}{\mu\varkappa_2}\frac{\partial p}{\partial t} \tag{2.8}$$

which gives the local relation between the changes in porosity and the pore pressure

$$m = m_0 - (1-m_0-\varepsilon)[\beta_s + (1-\varepsilon)/(K+G)](p-p_\infty) \tag{2.9}$$

if, as above, the reading is taken from the state of the medium at infinity.

The stress field is determined by Hooke's law (1.5) if use is again made of relation (2.3) and the boundary condition $\sigma = \sigma_\infty$ for the effective stress at infinity.

## 3. THE PLASTIC FLOW OF SAND

Suppose the effective radius of the active well $a$ is fairly small. This enables us to model the well with a point drain and to construct solutions by changing to the self-similar variable at the very beginning and, consequently, to preserve the non-linearity of the initial equations. Correspondingly, we consider that the drain is switched on at the instant $t = 0$ with constant rates of inflow of fluid $Q_f$ and sand $Q_s$. Rest conditions are satisfied at infinity $(r/a \to \infty)$, that is, the pressure, porosity and effective stresses are constant and equal to their initial values (when $t = 0$).

The zone of plastic flow is localized near the well. Within the zone, we shall neglect the elastic component of the deformations. Therefore, it is necessary to match the plastic flow velocity field and the elastic displacement field at the elastoplastic boundary $b$. This turns out to be possible on a moving boundary $b(t)$ which will move into the depth of the bed.

The usual continuity conditions

$$[\sigma_r] = 0, \quad [p] = 0, \quad [v] = 0, \quad [m] = 0 \qquad (r = b(t)) \tag{3.1}$$

must be satisfied on the elastoplastic boundary.

The position of the above-mentioned boundary is determined by failure condition (1.8), which the elastic stresses at its outer side must satisfy. Since, generally speaking, the material has disintegrated at the inner side of this boundary, its strength parameters are different and, consequently, the hoop stresses will now not be continuous on the boundary $r = b(t)$.

We will now extend the procedure for constructing a self-similar solution of the above problem. Correspondingly, we introduce the new variables

$$w\sqrt{t/\varkappa} = w^*, \quad v\sqrt{t/\varkappa} = v^*, \quad u/\sqrt{\varkappa t} = u^* \tag{3.2}$$

as functions of the self-similar variable $\xi$ which has been used previously, that is,

$$\xi = \frac{r}{\sqrt{\varkappa t}}, \quad \frac{\partial}{\partial t} = -\frac{1}{2t}\xi\frac{d}{d\xi}, \quad \frac{\partial}{\partial r} = \frac{1}{\sqrt{\varkappa t}}\frac{d}{d\xi}$$

Since the velocity of motion of the matrix is the time derivative of the displacement $v = du/dt$, we have

$$v^* = \frac{u^*}{2} - \frac{1}{2}\xi\frac{du^*}{d\xi} \tag{3.3}$$

in the new variables.

The boundary conditions must now correspond to the steady-state solution for the flow of an incompressible fluid together with incompressible particles of sand in the space of $\xi$. In this case, the porosity nevertheless changes by virtue of dilatancy effects or an elastic bulk deformation of the bed matrix itself.

We transpose the above mentioned sink intensities (for bulk flow rates) to the boundary $\xi = \xi_a = \text{const} = a/\sqrt{\varkappa t}$, that is,

$$Q_f = -2\pi ahwm \equiv -2\pi h\xi_a w^* m = \text{const}$$

$$Q_s = -2\pi ahv(1-m) = -2\pi h\xi_a v^*(1-m) = \text{const}$$

If these conditions are supplemented by a zero value of the effective stress on the wall of the well

$$\sigma_r = 0, \quad p = p_a \quad (r = a(t))$$

then the above-mentioned radius takes the meaning of the boundary of a cavity, in the sandy mass around the well, which grows with time.

As has been stated, the initial conditions and the conditions at infinity

$$\sigma_r = -(\Gamma - p_0), \quad p = p_0, \quad m = m_0 \quad (\xi = \infty)$$

are identical.

In calculations, these conditions will refer to a fairly large value of the radius $R(t)$ or to the value of $\xi_R$:

$$p = p_0 \equiv p_R, \quad m = m_0 \equiv m_R, \quad \sigma_r = -(\Gamma - p_R)$$

The value $\xi_R = 1$ was adopted in the calculations, corresponding to a real radius $R(t)$ which increases in proportion to $\sqrt{\varkappa t}$. Here, $\Gamma$ is the overall mining pressure.

In the new self-similar variables, the system of equations for the elastic zone has the form

$$p^* = \frac{p}{Y_0}, \quad \frac{dp^*}{d\xi} = -\frac{R^2\mu}{kY_0T} m(w^* - \upsilon^*) \tag{3.4}$$

$$\frac{dw^*}{d\xi} = \frac{1}{m}\left(\frac{\xi}{2} - w^*\right)\frac{dm}{d\xi} - \frac{w^*}{\xi}, \quad \left(\upsilon^* - \frac{\xi}{2}\right)\frac{dm}{d\xi} = (1-m)\left(\frac{\upsilon^*}{\xi} + \frac{d\upsilon^*}{d\xi}\right) \tag{3.5}$$

$$\frac{d\upsilon^*}{d\xi} = -\frac{\upsilon^*}{\xi}\frac{\nu}{1-\nu} - \frac{1-2\nu}{2(1-\nu)B}\xi\left(\frac{d\sigma_r^*}{d\xi} - \varepsilon\frac{dp^*}{d\xi}\right) \tag{3.6}$$

$$\frac{d\sigma_r^*}{d\xi} = \frac{dp^*}{d\xi} + \frac{2B}{\xi^2}\upsilon^*, \quad \sigma_r^* = \frac{\sigma_r}{Y_0}, \quad B = \frac{2G}{Y_0} \tag{3.7}$$

Here, (3.4) is Darcy's law, which includes the rate of displacement of the matrix, (3.5) are the transformed balances of the incompressible masses, (3.6) is the relation between the rate of displacement and the displacement itself, which takes account of Hooke's law in differential form, and (3.7) is the equilibrium equation for the matrix. A dimensionless notation with respect to the initial cohesion and the relaxation time $T = R^2/\varkappa$, which is determined by the piezoconduction $\varkappa$ of the bed, has been used.

The plasticity criterion

$$\sigma_r^*(+) = \frac{1}{1-N}\left(1 + \frac{2BN\upsilon^*(-)}{\xi}\right) \tag{3.8}$$

is satisfied on the elastoplastic boundary ($\xi = \xi_b$) as well as the continuity condition

$$[p^*] = 0, \quad [\sigma_r^*] = 0, \quad [m] = 0, \quad \upsilon^*(-) = \frac{1}{2}u^*(+) - \frac{1}{2}\xi\frac{du^*}{d\xi}(+) \tag{3.9}$$

The plus sign corresponds to the elastic side of the boundary and the minus sign to the plastic side. Note that (3.8) is a consequence of criterion (1.8), elastic relations (1.5), and self-similar relations (3.2) and (3.3). Condition (1.8), which can be represented in the form

$$\sigma_\theta = N\sigma_r - Y_0, \quad N = \frac{\theta_\sigma + \alpha}{\theta_\sigma - \alpha}, \quad Y_0 = \frac{Y}{(\theta_\sigma - \alpha)} \tag{3.10}$$

is satisfied inside the plastic flow zone.

The solution is characterized by a jump in the hoop stress $\sigma_\theta$ which is caused by the replacement of the initial cohesion $Y = Y(+)$ by its residual value $Y_r = Y(-)$. Changes in the porosity can also be taken into account here, see [9].

Darcy's law (3.4) and the mass balance (3.5) are satisfied in the plastic flow zone but, instead of elastic relation (3.6), a dilatancy condition and an equilibrium equation of a different form are introduced. These are as follows:

$$\frac{d\upsilon^*}{d\xi} = -n\frac{\upsilon^*}{\xi}\left(n = \frac{1+\theta\Lambda}{1-\theta\Lambda}\right), \quad \frac{d\sigma_r^*}{d\xi} + \frac{(1-N)\sigma_r^* + Y_r/Y}{\xi} - \frac{dp^*}{d\xi} = 0 \tag{3.11}$$

The permeability of the medium as well as the porosity undergo irreversible changes. For the calculation, we use the well-known Kozeny–Carman formula

$$\frac{k_0}{k} = \frac{(1-m)^2 m_0^3}{m^3(1-m_0)^2}$$

However, in some versions of the calculation in this paper it can be assumed to be a constant quantity.

## 4. CALCULATION OF THE SAND PRODUCTION BY THE LIQUID FLOW

The system of ordinary differential equations (3.4)–(3.11) was solved numerically using the Runge–Kutta method as modified by Ralston. Here, the Hamming "predictor–corrector" procedure was used.

The five functions $p^*$, $\sigma_r^*$, $w^*$, $v^*$ and $m$ were the required functions since the relation $v = du/dt$ enables one to eliminate the displacement. Criterion (3.1) was checked at each step of the calculations. The rate of displacement on the outer contour was chosen such that the condition $\sigma_r = 0$ was satisfied, when $a = \xi_a\sqrt{\varkappa t}$, on the contour of the cavern (the void) which is formed accompanying the sand production. The calculated outputs of sand and liquid were determined from the velocity field.

The following set of parameters was used in the calculations: the viscosity $\mu = 10^{-2}$ Pa·s = 10 centipoise, the shear modulus $G = 100$ MPa, Poisson's ratio $\nu = 0.3$, the initial porosity $m_0 = 0.2$, the initial permeability $k_0 = 10^{-13}$ m$^2$ = 100 millidarcy, the initial cohesion $Y = 3$ MPa, the coefficient of internal friction $\alpha = 0.5$, the bed height $h = 10$ m, the pressure on the outer contour $p_R = 6$ MPa ($r = R$) and the effective stress on the outer contour $\sigma_r(R) = -\Gamma = -10$MPa ($r = R$).

The residual cohesion $Y_r$ and the rate of dilatancy $\Lambda$ were varied in the calculations ($\Lambda$ was varied from −0.5 to 0.5 in versions 1 and 5 of Table 1; in all of the other versions $\Lambda = -0.1$). Note that versions 5–7 are considered in Section 5.

The compressibility of the grains was assumed to be negligibly small, that is, $\varepsilon = 0$ (the effect was small in test calculations, when it was assumed that $\varepsilon = 0.1$). In the computerized implementation of the solution, the compression was assumed to be positive (as is assumed in mining mechanics).

The relaxation time of the bed was defined as $T = R^2/\varkappa$ and the expression $\varkappa = \varkappa_1$ was used for the piezoconduction coefficient. Specifically, it was equal to 0.0032 m$^2$/s in all the calculations.

The liquid flow rate was negative (a flow to the well, i.e. the seepage rate is directed towards a reduction in the radius) and equal to $Q_f = -5{,}6\times10^{-4}$ m$^3$/s.

The influx of sand $Q_s$ was determined in calculations, the results of which are shown in the figures (the solid lines) and in Table 1 (versions 1–4). The dependence of the stresses, the porosity and the reduced velocities and the pore pressure on the self-similar variable is shown in Figs 1, 2 and 3 respectively. They are all presented for the case of version 1 with the least sand production (in the case when the cohesion of the still intact matrix and the cohesion in the plastic flow of its fragments, that is, of the sand, are equal). The calculated curves in Figs 1–4 correspond to a value of $\Lambda = -0.1$.

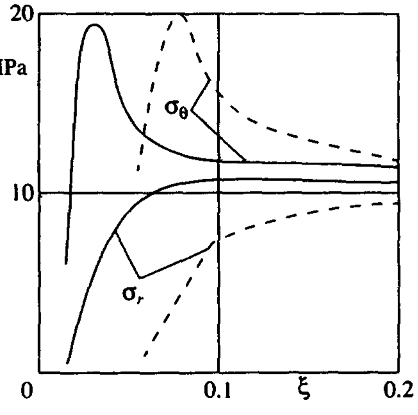


Fig. 1

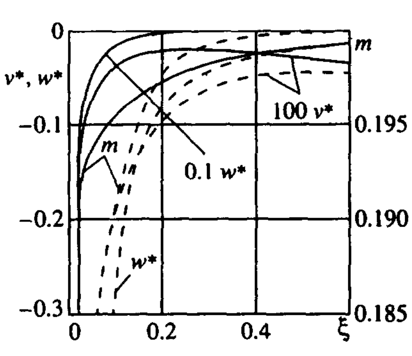


Fig. 2

Table 1

| Version | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $Y_r$, MPa | 3.0 | 1.5 | 1.2 | 0.9 | 5 | 2.5 | 1.5 |
| $Q_s$, m³/s | $4.0\times10^{-6}$ | $7.7\times10^{-6}$ | $1.9\times10^{-5}$ | $3.4\times10^{-4}$ | 0.0144 | 0.0217 | 0.045 |
| $\Pi = 100Q_s/Q_f$ | 0.7 | 1.38 | 3.5 | 60.7 | 1.61 | 2.42 | 5 |
| $b/a$ | 1.73 | 2.42 | 3.03 | 3.85 | 1.40 | 1.87 | 2.95 |
| $b/R$ | 0.0266 | 0.0382 | 0.0618 | 0.271 | 0.0672 | 0.085 | 0.128 |

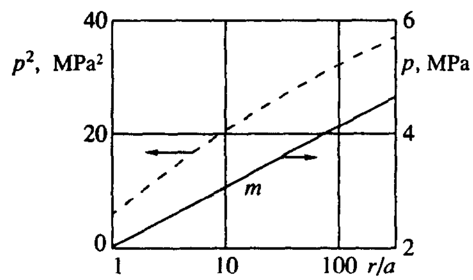


Fig. 3

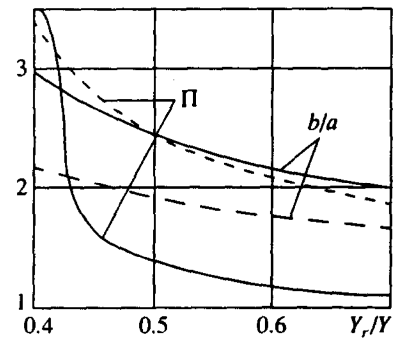


Fig. 4

As might have been expected, the profile of the radial stresses is continuous and smooth. At the same time, the hoop normal stress distribution $\sigma_\theta$ has a discontinuity on the elastoplastic boundary (Fig. 1) and, when the value of the cohesion is changed on this boundary, these stresses change suddenly.

The rates of displacement of the phases are negative, since they are directed towards the origin of coordinates, that is, towards the well. In this case, the velocity profile of the solid phase has a maximum. In logarithmic coordinates, the pressure distribution is found to be practically linear over a significant range (from 0 to 2.4). This means that the radius of the contour which has to be introduced into the *steady-state solutions* [6, 7] is approximately equal to $250a$ in all of the versions of the initial to the residual cohesion ratio which have been considered here. At the same time, the plastic zone is small (if no catastrophic flow sets in as in versions 3 and 4).

The intensity of the sand production $\Pi = 100\ Q_s/Q_f$ is mainly due to the cohesion in the zone of plastic flow of the sand. The changes in the magnitude of Π are therefore comparable with the cohesion in the flow of sand but relative to the cohesion of the intact matrix (Fig. 4 and Table 1).

It can be seen that the sand constitutes one hundredth part by volume of the extracted liquid (petroleum or water) and the outer radius of the zone of plastic flow of the sand is only two to three times greater than the radius $a$ of the open well (or of the cavern which is formed). If the cohesion in the flow of the mass of sand decreases by a factor of three or more compared with the initial value, the intensity of the sand production increases sharply.

The residual reduction in the porosity in the plastic flow zone (in the case of a negative value for the rate of dilatancy) far exceeds its changes in the elastic zone. In the case of positive dilatancy in the plastic zone, disintegration occurs which manifests itself in an increase in the sand production by a factor of almost three (see Fig. 5, version 1).

The rates of increase of the radius of the cavern and the radius of the zone of plastic flow of the sand are determined by the piezoconduction coefficient. In the case of its value mentioned above, the cavern

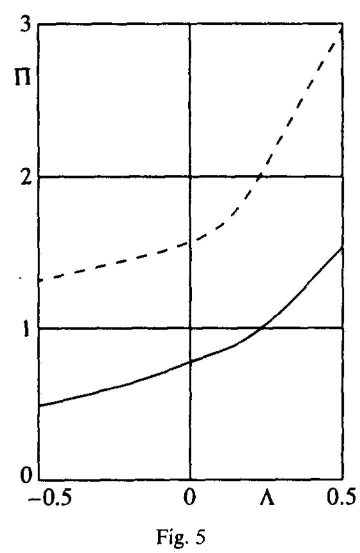


Fig. 5

reaches a radius of 0.2 m after 1 hour (in versions 1 and 2). Note the fact that, in these calculations, no account has yet been taken of the hardening of the sand mass when there is a reduction in the porosity and a corresponding change in the rate of dilatancy. Note also that the changes in the permeability (according to Kozeny's formula) are not too great in the case of the sand production. To an equal extent, making allowance for the discontinuity in the porosity (by 0.05 in version 2) which accompanies changes in cohesion at the boundary of the plastic flow did not prove to be very important (the sand production increased by 20% and the radius of the plastic flow zone by 4%).

## 5. CALCULATION OF THE SAND PRODUCTION BY THE GASFLOW

In the case of the sand production by a gas well, the direct proportionality of the density and pressure changes the mass balance for the fluid

$$\frac{dw^*}{d\xi} = \frac{1}{m}\left(\frac{\xi}{2} - w^*\right)\frac{dm}{d\xi} - \frac{w^*}{\xi} + \left(\frac{\xi}{2} - w^*\right)\frac{1}{p^*}\frac{dp^*}{d\xi}$$

The remaining equations retain the same form as in Section 4.

The following data were used for the calculations: the viscosity $\mu = 2 \times 10^{-5}$ Pa·s, $G = 100$ MPa, Poisson's ratio was 0.3, the permeability $k_0 = 10^{-13}$ m$^2$ and the initial cohesion $Y = 5$ MPa; the residual cohesion $Y_r$ and the rate of dilatancy $\Lambda$ were varied; the porosity $m_0 = 0.2$, $\varepsilon = 0$, $p_R = 6$ MPa, $\sigma_R = -10$ MPa, the friction coefficient was 0.5, the bed thickness $h = 10$ m, the piezoconduction coefficient was 1.6 m$^2$/s and $Q_j = 0.896$ m$^3$/s = 77400 m$^3$ per day.

The results are represented by the dashed curves in Figs 1–5 and in Table 1 (versions 5–7). The calculated curves in Figs 1–4 correspond to a value of $\Lambda = -0.1$. The results of calculations of the sand production as a function of the rate of dilatancy are shown in Fig. 5 for the version when the residual cohesion is equal to the initial cohesion.

It is clear that the stress distribution along the radius in this case does not change qualitatively compared with the case of sand production by a liquid flow. However, the velocity profile which is generated by the gas flow no longer has an internal maximum (Figs 1 and 2). The range of the logarithmic distribution of the square of the pore pressure along the radius (Fig. 3), which corresponds in practice to the steady influx of gas into the undeformed bed, is somewhat contracted (in $\xi$ coordinates down to a value of 1.3). This means that the feeding contour radius in the case of steady flow is of the order of $20a$.

The sand production by the gas flow proves to be three times more intense than by a liquid flow (Figs 4 and 5), although the plastic zone is of the same order of magnitude (in radii of the cavern). It can be assumed that such a substantial enhancement of sand production is due to the significantly greater gradients of the pore pressure in the neighbourhood of a gas well. In reality, according to known results [12], the sand production occurs from the small weakest layers or by means of individual wormholes [13] in the bulk of the unfractured bed. The latter are formed due to a loss of stability of the plastic flow of a Coulomb medium [2.5].

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